Environment and Planning B: Planning and Design 1998, volume 25, pages 849 – 863

Comparing connectivity in landscape networks

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Received 30 August 1997; in revised form 18 June 1998

Abstract. Existing parameters that measure the connectivity of elements in landscape networks are dependent upon the size of the network, defined as the number of locations in it. These parameters are unsuitable for comparing the connectivity in networks that have different numbers of locations. The objective of this paper is to extend the existing parameters, so that the degree of connectivity of elements in a variety of networks can be compared. These parameters can be used to test relationships between the connectivity of locations and their function in the region and to compare these relationships across several locations in different regions as well as in one location in time. In simulations, we show that the size of the graph affects the values of the parameters derived. However, in contrast to the existing parameters, the degree of connectivity of elements in differently sized networks can be usefully compared. The parameter based on shortest-weighted paths can also differentiate between differently spaced networks.

1 Introduction

One of the primary concerns in spatial analysis is the position of locations in a region relative to one another, because locations adopt their roles in the region as a function of their connectivity to the system as a whole (Cantwell and Forman, 1993; Haggett et al, 1977; Hillier and Hanson, 1984; Lowe and Moryadas, 1975; Taaffe and Gauthier, 1973; Tinkler, 1977). Differences occur in the connectivity of locations, and hence in their function, owing to differently spaced and sized systems.

We define connectivity as a property of locations to maintain spatial or functional relationships with other locations in terms of flows of entities (materials, energy, information, people, animals, etc). This definition embraces other terms such as accessibility of locations, so long as they emphasize spatial characteristics that direct relationships between locations. The locations in a region constitute landscape networks as a result of these relationships. Graph theory provides parameters to quantify the degree of connectivity in such networks. These parameters have a long history and have been widely applied in geographical research, especially for analyzing communication and transportation networks (Allen et al, 1993; Garrison and Marble, 1965; Ingram, 1971; Mackiewicz and Ratajczak, 1996; Shimbel, 1953; Taaffe and Gauthier, 1973). The values of these parameters are dependent upon the size of networks (we define the size of a network as the number of locations in it.). These parameters are thus unsuitable for comparing the connectivity in networks that have different numbers of locations (Allen et al, 1993; Teklenburg et al, 1993).

Our objective is to extend the existing parameters, so that the degree of connectivity of elements in a variety of networks can be compared. We do not apply these parameters to explain certain observed patterns in landscapes. The background to this paper is our study of fragmentation effects in animal populations, where we dealt with locations in different (subsets of) networks whose relative position in the network may determine colonization processes. As we wish to explain the relationships between the relative position of the elements and colonization processes, the relative positions should be measured so that they can be compared. In our opinion, however, the issue of comparing the connectivity in landscape networks can also be relevant to other fields in spatial analysis.

We address two questions.

(1) Does the size of networks affect the degree of connectivity measured by the parameters?

(2) Does variation in the parameters concur with the variation in the spatial configuration of the network?

2 Comparing the degree of connectivity of locations

Connectivity parameters can be used to assess the function of a location. For example, for economic purposes the parameters can be used to determine whether a location is an element with a high degree of inward and outward traffic in the transportation network or is a centre for economic activities (Allen et al, 1993; Dupuy and Stransky, 1996). These parameters can also be applied in studies with other objectives. For example, locations may act as a point of attraction in a network of tourist movements (van der Knaap, 1997) or as a source for dispersal of species in the region (Hanski and Gilpin, 1997). To test and compare these relationships across several locations in different regions as well as in one location in time, the degree of connectivity of elements should be measured and compared with the degree of connectivity measured in other networks (figure 1).

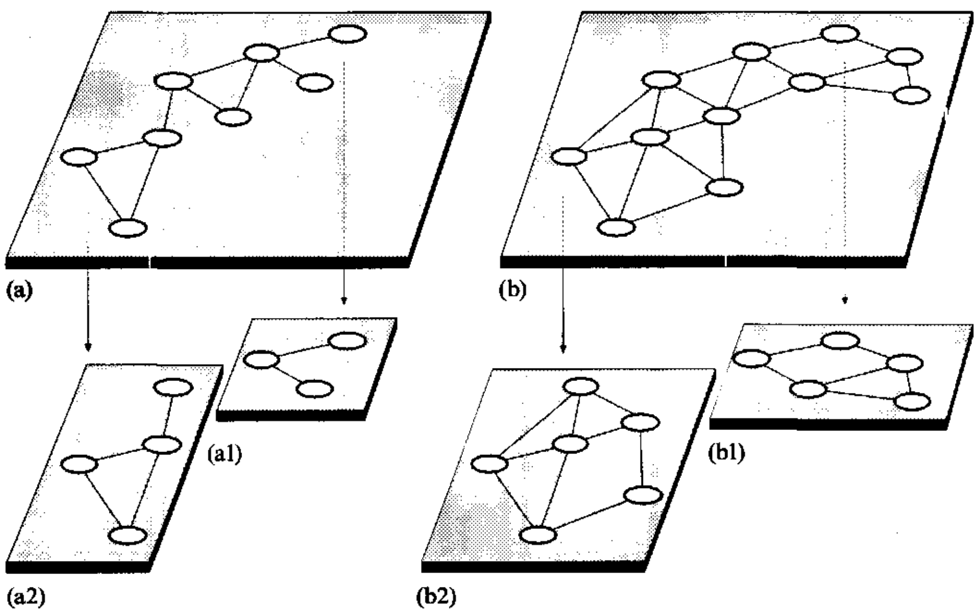


Figure 1. Measuring the degree of connectivity of elements in two networks (a) and (b) should enable us to compare the elements of these networks with each other. The elements of the nonconnected subsets (a1) and (a2) or (b1) and (b2) of the decomposed networks (a) and (b) should also be compared. It is obvious that network (b) and its subsets are more complex than network (a) and its subsets, and that most elements in network (b) and its subsets have a higher degree of connectivity.

A special case in such comparisons is when the elements belong to different subsets in one network (figure 1). Networks decomposed into different subsets of elements can be considered as disconnected or nonconnected (Wilson and Watkins, 1990). It may be relevant to test the degree of connectivity of elements in different subsets, for example, when the accessibility of a location by different types of transportation systems needs to be determined (for example, either by bus, by train, or a combination of both; Tinkler, 1977), or in the analysis of the position of large cities in the European highway network related to their position as elements in the national network (Dupuy and Stransky, 1996). Subsets of elements also occur in analyses where relationships between certain sets of elements are restricted, for example, in the study of effects of habitat fragmentation in population ecology (Hanski and Gilpin, 1997; Taylor et al, 1993) where population survival in subsets of habitat patches depends upon the number and spacing of the elements in the subset. The function of an element in a subset may depend upon relationships with all other elements in the subset.

Relevant questions in spatial analysis concern whether differences in the degree of connectivity of locations in different (subsets of) networks are related to their function. To address such questions where comparison among elements in different (subsets of) networks comes up, we extend the existing parameters. For those who are less familiar with graph theory, we first introduce the matrix-based approach to measure connectivity of elements in networks.

3 Matrix-based approach for connectivity analysis

3.1 Definition of distance

Matrix-based parameters measure the degree of connectivity of network elements as a function of the number of (direct and indirect) neighbouring elements and the distance between these elements. Connectivity may be defined in various ways. The main distinction between matrix-based parameters depends on the use of space to define distance. Topological and geometric approaches exist. In the topological approach, the presence or absence of an edge between vertices is considered. In this context, the cardinality t_{ij} of a path between two vertices in a graph is the number of edges between these vertices. In the geometric approach, positive numeric weights, w_{ij}, are assigned to each edge in the graph, for example, the costs of movement, Euclidean distance, time required to move between elements, amount of flow, etc. The weights offer additional information about the relationships between elements. In the context of this paper, we use high values of w_{ij} between two vertices to indicate a low degree of connectivity, and vice versa.

Two matrix-based parameters can be distinguished for both the topological and the geometric approaches: those for direct connections and those for shortest paths. These parameters can be applied to nondirected and directed graphs that are either connected or nonconnected.

3.2 Connectivity in nondirected graphs

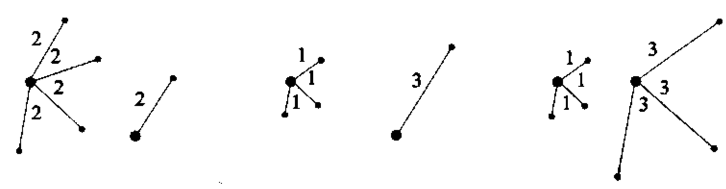
First, we consider a nondirected graph $G(V, E)$ with vertex set $V(G)$, $V(G) = \{v_1, v_2, \ldots, v_n\}$, with n vertices, and edge set $E(G)$, $E(G) = \{e_1, e_2, \ldots, e_m\}$, without self-loops and multiple edges. The topological connectivity matrix $\mathbf{C}$, $\mathbf{C} = (c_{ij})$, of $G(V, E)$ is defined as: $c_{ij} = 1$, if a direct connection between the vertices v_i and v_j exists in $G(V, E)$; $c_{ij} = 0$, otherwise. The geometric equivalent of $\mathbf{C}$ is the weighted-distance matrix $\mathbf{P}$, $\mathbf{P} = (p_{ij})$, defined as: $p_{ij} = w_{ij}$, if a direct connection between v_i and v_j exists in $G(V, E)$; $p_{ij} = 0$, otherwise. The weighted length p_{ij} of direct edges of v_i provides a measure for the geometric proximity of all neighbouring v_j. Matrix $\mathbf{D}$, $\mathbf{D} = (d_{ij})$, is the shortest-path distribution between vertices: d_{ij} is the cardinality t_{ij}

of the shortest path between v_i and v_j. The diameter δ of the graph is defined as $\delta = \text{maximum}\ \{d_{11}, d_{12}, \dots, d_{nn}\}$. Matrix **S**, $\mathbf{S} = (s_{ij})$, provides the shortest-weighted paths between pairs of vertices: s_{ij} is the cumulative edge length w_{ij} of the shortest path between v_i and v_j. To obtain **S**, an heuristic algorithm is applied (Taaffe and Gauthier, 1973; Tinkler, 1977). As a result, the paths generated are not necessarily the shortest paths, but approximations. We define the geometric diameter σ of the graph as $\sigma = \text{maximum}\ \{s_{11}, s_{12}, \dots, s_{nn}\}$.

The four matrix types can measure three characteristics of networks and their elements: the relative position of elements, their relative importance, and the network dispersion. Each matrix type provides a quantitative measure for the relative position of elements in terms of highly connected elements versus badly connected elements in the network. This is obtained by the vector that sums the elements of each row or column in the matrix. For nondirected graphs, the vector $\boldsymbol{c}$ can be obtained by multiplying matrix **C** with the vector $\boldsymbol{l}$:

$$\boldsymbol{c} = \mathbf{C}\boldsymbol{l}, \tag{1}$$

where $l_i = 1$, $i = 1, 2, \dots, n$. The elements c_i of $\boldsymbol{c}$ provide a measure for the relative position of v_i. For **D**, **P**, and **S**, the corresponding vectors $\boldsymbol{d}$, $\boldsymbol{p}$, and $\boldsymbol{s}$ can be obtained by an analogous process to that shown in equation (1). For the elements of vector $\boldsymbol{p}$ we advocate a modification. One may expect that low values of p_i refer to v_i with a high degree of connectivity. However, if more than one edge is connected to v_i, p_i can be misleading. For example, we assumed that a vertex connected with four edges and $w_{ij} = 2$ has a degree of connectivity four times higher than that of a vertex connected with one edge and $w_{ij} = 2$ [figure 2(a)]. The same reasoning was applied for a vertex connected with three edges and $w_{ij} = 1$, which has a degree of connectivity three times higher than that of a vertex connected with one edge and $w_{ij} = 3$ [figure 2(b)]. Therefore, each p_i should be modified. As connectivity declines with increasing distance w_{ij}, we used the reciprocal of w_{ij} to obtain a consistent measure. This is analogous to the commonly used population potential models (Pooler, 1987). Matrix $\mathbf{P}'$ is then defined as: $p'_{ij} = 1/w_{ij}$, if a direct connection between v_i and v_j exists in $G(V, E)$; $p'_{ij} = 0$, otherwise. Vector $\boldsymbol{p}'$ can be calculated in the same way as shown in equation (1). Figure 2 shows the comparison between c_i, p_i, and p'_i for different graphs.



	(a)		(b)		(c)	
c_i	4	1	3	1	3	3
p_i	8	2	3	3	3	9
p'_i	2	0.5	3	0.333	3	1
	4× ←		9× ←		3× ←	

Figure 2. Paired comparison of network patterns illustrating the modification of parameter p_i. The degree of connectivity of the large vertices i (●) is considered. The c_i, p_i, and modified p'_i values are given. High values of c_i and p'_i correspond to highly connected vertices. The arrows indicate the extent to which, in a given pair, the left vertex has a higher degree of connectivity (measured by p'_i) than the right vertex. It should be noted that comparisons between more complex patterns are difficult to interpret in simple terms.

The elements d_i in $\boldsymbol{d}$ sum the shortest paths d_{ij} between v_i and all other vertices. For a given vertex v_i, each d_{ij} of length r $(1 \leqslant r \leqslant \delta)$ occurs with a particular frequency $f_{i,r}$ (James et al, 1970). This frequency distribution is a finite discrete set. Another formulation of d_i is thus

$$d_i = \sum_{r=1}^{\delta} f_{i,r} r, \quad i = 1, 2, \ldots, n. \tag{2}$$

We used this alternative formulation to derive our intended parameters.

The elements s_i in $\boldsymbol{s}$ sum the shortest-weighted paths s_{ij} between v_i and all other vertices. This parameter has been suggested as a suitable measure for analyzing variation in the spatial configuration of networks (Taaffe and Gauthier, 1973). Because the frequency distribution $f_{i,u}$ of shortest-weighted paths s_{ij} of length u $(0 < u \leqslant \sigma)$ for a given vertex v_i to all v_j is a finite discrete set, s_i can be represented by

$$s_i = \sum_{u>0}^{\sigma} f_{i,u} u, \quad i = 1, 2, \ldots, n. \tag{3}$$

Because s_{ij} is continuous, $f_{i,u}$ is often 1.

High values of c_i and p_i' correspond to a high degree of connectivity. For d_i and s_i, an inverse relationship exists: v_i with the lowest d_i or s_i has the highest degree of connectivity. A hierarchy embodying the relative importance of elements can be obtained by ranking the connectivity values. The network dispersion $\hat{c}$ of matrix $\mathbf{C}$ measures the connectivity or the compactness of the whole network (Shimbel, 1953). It can be calculated by multiplying the transpose of $\boldsymbol{c}$ with $\boldsymbol{l}$:

$$\hat{c} = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{l}. \tag{4}$$

The network dispersion $\hat{p}'$, $\hat{d}$, and $\hat{s}$ can be obtained by using their corresponding vectors $\boldsymbol{p}'$, $\boldsymbol{d}$, and $\boldsymbol{s}$ in the same way as shown in equation (4).

3.3 Connectivity in directed graphs

We can also consider a directed graph or digraph $D(V, A)$ with vertex set $V(D)$, $V(D) = \{v_1, v_2, \ldots, v_n\}$, with n vertices, and arc set $A(D)$, $A(D) = \{a_1, a_2, \ldots, a_m\}$, without self-loops and multiple arcs. Arcs are directed edges. For digraph $D(V, A)$, the same matrix-based parameters can be applied. However, in contrast to nondirected graphs, the matrices of digraphs are not necessarily symmetric about the principal diagonal. The rows of these matrices represent the origin locations for the connecting relationships, and the columns the destination locations.

For matrix $\mathbf{C}^D$ as derived from digraph $D(V, A)$, two vectors are distinguished. The vector $\boldsymbol{c}^{\mathrm{out}}$ can be obtained by multiplying matrix $\mathbf{C}^D$ with the vector $\boldsymbol{l}$:

$$\boldsymbol{c}^{\mathrm{out}} = \mathbf{C}^D \boldsymbol{l}. \tag{5}$$

The vector $\boldsymbol{c}^{\mathrm{in}}$ is given by

$$\boldsymbol{c}^{\mathrm{in}} = \boldsymbol{l}^{\mathrm{T}} \mathbf{C}^D, \tag{6}$$

in which $\boldsymbol{l}^{\mathrm{T}}$ is the transpose of vector $\boldsymbol{l}$. The elements c_i^{out} provide a measure for the relative position of v_i concerning outward relationships; the elements c_i^{in} provide measures for the inward relationships. For $\mathbf{D}^D$, $\mathbf{P}^D$, and $\mathbf{S}^D$ as derived from digraph $D(V, A)$, the corresponding vectors can be obtained as demonstrated in equations (5) and (6). Here, unless otherwise specified, we will focus on nondirected graphs, without self-loops and multiple edges.

3.4 Connectivity in nonconnected graphs

We advocated that if g different graphs are considered we should be able to compare the degree of connectivity of the elements among different graphs $G_j(V, E)$ and $G_k(V, E)$, where j and k are elements of g. When a network is decomposed into non-connected subsystems, the graph $G(V, E)$ of this network consists of several disjointed subgraphs. In graph theory, nonconnected graphs are distinguished as graphs with specific properties. In our study of fragmentation effects on animal populations, we dealt with nonconnected networks (van Langevelde et al, 1998). We argued that comparison among different graphs is equivalent to the comparison among different subgraphs in a nonconnected graph (see figure 1).

A graph $G(V, E)$ is connected if there is at least one direct or indirect path between any pair of vertices; otherwise it is nonconnected. Graph $G(V, E)$ of a nonconnected network is the union of g subgraphs. We define a subgraph $G_k(V, E)$ as the kth component of $G(V, E)$. The vertex subset $V(G_k)$ with n_k vertices and edge subset $E(G_k)$ of $G_k(V, E)$ of a nonconnected network are defined as

$$V(G_k) \subset V(G) := \{V(G_1), V(G_2), \dots, V(G_g)\},$$

$$E(G_k) \subset E(G) := \{E(G_1), E(G_2), \dots, E(G_g)\},$$

and

$$G_k(V, E) \subset G(V, E) := \{G_1(V, E), G_2(V, E), \dots, G_g(V, E)\}.$$

In nonconnected graphs

$$V(G_j) \cap V(G_k) = \{\phi\} \quad \text{and} \quad E(G_j) \cap E(G_k) = \{\phi\}, \quad \text{for } j \neq k.$$

The vector $\boldsymbol{c}_k$ of subgraph k can be obtained by multiplying $\mathbf{C}$ by the vector $\boldsymbol{l}_k$:

$$\boldsymbol{c}_k = \mathbf{C}\boldsymbol{l}_k, \quad k = 1, 2, \dots, g, \tag{7}$$

where $l_{i,k} = 1$ for $v_i \in V(G_k)$, and $l_{i,k} = 0$ for $v_i \notin V(G_k)$. Then for $v_i \in V(G_k)$, $c_{i,k} \neq 0$, unless $n_k = 1$. The network dispersion $\hat{c}_k$ of each subgraph k is obtained from

$$\hat{c}_k = \boldsymbol{c}_k^{\mathrm{T}} \boldsymbol{l}_k, \quad k = 1, 2, \dots, g. \tag{8}$$

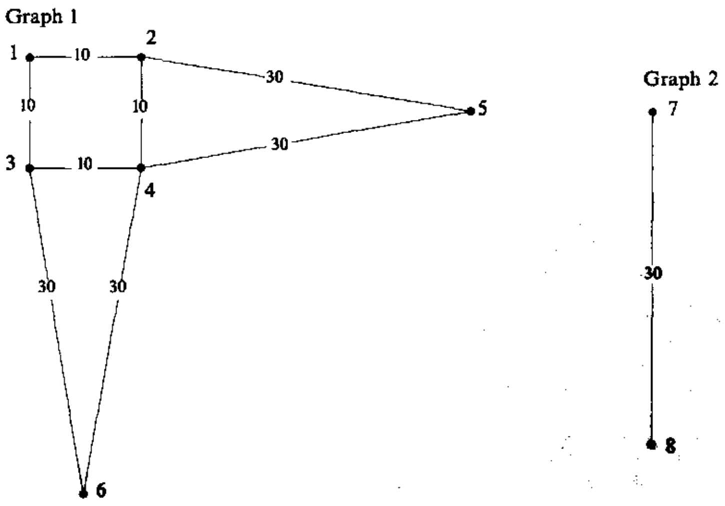
The vectors $\boldsymbol{p}'_k$, $\boldsymbol{d}_k$, and $\boldsymbol{s}_k$ and the network dispersion $\hat{p}'_k$, $\hat{d}_k$, and $\hat{s}_k$ can be calculated as shown in equations (7) and (8).

In the next section, we derive parameters that are suitable for comparing the degree of connectivity of elements in g different graphs where each graph $G_k(V, E)$ consists of vertex set $V(G_k)$ with n_k vertices and edge set $E(G_k)$. The parameters should also allow comparison of the degree of connectivity of elements in g subgraphs of a nonconnected graph where each subgraph $G_k(V, E)$ consists of vertex subset $V(G_k)$ with n_k vertices and edge subset $E(G_k)$.

4 Parameters to compare the degree of connectivity in different networks

4.1 Problem with comparing the degree of connectivity

Figure 3 illustrates the problem of using either $\boldsymbol{d}$ or $\boldsymbol{s}$ for comparing the degree of connectivity of elements in different networks. The d_i and s_i values of the vertices are listed in the figure. For each graph, low values of both d_i and s_i represent the best-connected vertices. However, the values returned for d_i and s_i reveal that v_4 is better connected than v_7. This is a counterintuitive notion when the size and spacing of the networks are considered. Elements in large networks are likely to have a higher degree of connectivity than elements in smaller networks. Also, when the distances between vertices increase, the degree of connectivity of the vertices is assumed to decrease.



v_i	d_i	Order	s_i	Order
Graph 1				
1	8	5	120	4
2	7	2.5	110	2.5
3	7	2.5	110	2.5
4	6	1	100	1
5	8	5	200	5.5
6	8	5	200	5.5

$\hat{d}_1 = 44, \quad \hat{s}_1 = 840$

v_i	d_i	Order	s_i	Order
Graph 2				
7	1	1.5	30	1.5
8	1	1.5	30	1.5

$\hat{d}_2 = 2, \quad \hat{s}_2 = 60$

Figure 3. Two graphs ($k = 1, 2$) with values of the connectivity parameters d_i and s_i per vertex v_i, the hierarchy numbers (order) of the vertices per graph related to d_i and to s_i, and the network dispersion $\hat{d}_k$ and $\hat{s}_k$ of the two graphs. Low values of d_i and s_i correspond to highly connected vertices. For both d_i and s_i, the best connected vertex has the lowest hierarchy number.

Two methods correct for the size bias of different networks. Allen et al (1993) argue that the normalized mean spatial separation E_k among the set of vertices in graph k overcomes the size effects; E_k of graph k is obtained by

$$E_k = \frac{1}{n_k(n_k - 1)} \sum_{i,j=1}^{n_k} s_{ij} \, .$$

The value E_k converges quickly to a stable value as the size n_k of the vertex set $V(G_k)$ increases. Teklenburg et al (1993) propose a standardization of their integration measure by using complete bipartite graphs. They show that its value is independent of the size of the vertex set $V(G_k)$. This method is used only in the topological approach (Teklenburg et al, 1993).

Both methods advocate a standardization of the network dispersion that relates to the degree of connectivity of the network as a whole. They place networks on a more or less fixed range of normalized degrees of connectivity. This range goes from completely connected networks with the highest degree of connectivity through to branching networks with the lowest degree of connectivity. Completely connected networks always have the same normalized value for the network dispersion, regardless of the size of the set $V(G_k)$. However, these methods neglect our assumption that locations in large landscape networks (with numerous elements) function differently from locations in small networks (with a small number of elements) and fail to meet our objective to

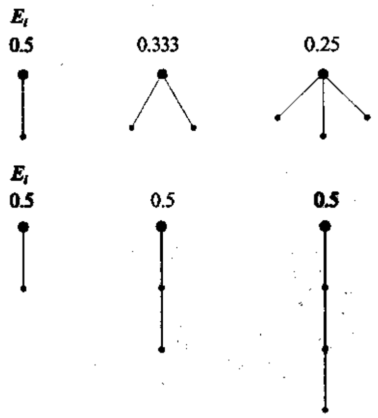


Figure 4. Calculation of the values for the normalized mean spatial separation E_i for vertex v_i based on equation (9). In the six graphs, $w_{ij} = 1$. The degree of connectivity of the large vertices i (•) is considered. Low values of E_i correspond to highly connected vertices. In the upper three graphs, E_i decreases. However, one should expect that the degree of connectivity of the vertex indicated in the first graph is twice the degree of the vertex in the second graph, and so on. In the lower three graphs, the degrees of connectivity of the indicated vertices do not differ in contrast to what can be expected when the shortest paths to all vertices are considered.

measure the connectivity of the individual elements. When applied to individual vertices, the measure proposed by Allen et al (1993) is calculated as

$$E_i = \frac{1}{n_k(n_k - 1)} \sum_{j=1}^{n_k} w_{ij} . \tag{9}$$

In figure 4, E_i is shown for the vertices v_i indicated by large dots. It appears that E_i cannot differentiate between vertices in differently sized graphs.

When vertices in subgraphs of nonconnected networks are considered, another problem arises if the degree of connectivity of elements in different subgraphs is compared. The parameters d_i and s_i measure shortest paths from v_i to all others. In order to calculate $\boldsymbol{D}$ and $\boldsymbol{S}$, d_{ij} and s_{ij} are assigned values of infinity if there is no path between v_i and v_j (Taaffe and Gauthier, 1973). This will not occur in connected graphs. For nonconnected graphs, the values of infinity in the matrices will dominate the connectivity values in $\boldsymbol{d}$ and $\boldsymbol{s}$. For these reasons we advocate a modification of the connectivity vectors $\boldsymbol{d}$ and $\boldsymbol{s}$ for the influence of the size n_k of the vertex set $V(G_k)$.

4.2 Topological approach

For d_i to be a suitable measure for comparing the degree of connectivity of vertices in a single graph, a prerequisite for modification is that the relative importance of vertices within each graph be constant. The d_i values should be modified for the influence of n_k because we assumed that vertices in a large graph k have a higher degree of connectivity than vertices in a smaller graph l ($n_k > n_l$). However, the opposite can also be true. Vertices with low c_i values within a large graph (such as a branching network) will have a lower degree of connectivity compared with vertices in a small, completely connected graph. We therefore introduce critical points. For what conditions do vertices within a small graph k have a higher degree of connectivity than vertices within a large graph l?

The effect of size on the connectivity parameters is ambiguous. How can the problem of comparing d_i values of vertices in differently sized graphs be resolved? As is shown by equation (2), d_i is based on the summation of $f_{i,r}$ multiplied by r. If more vertices are added to the vertex set $V(G_k)$, then either the maximum value of r will increase, the frequency $f_{i,r}$ for high values of r will increase, or both will increase.

Therefore the value of d_i increases when the size n_k of the set $V(G_k)$ is increased. For conformance with the calculation of p_i', we used the reciprocal of r to obtain a modification of the influence of r, and therefore of the size n_k of $V(G_k)$, in d_i. The modified d_i' value can be calculated by

$$d_i' = \sum_{r=1}^{\delta} \frac{f_{i,r}}{r}, \quad i = 1, 2, \ldots, n. \tag{10}$$

High values of d_i' represent highly connected vertices. The network dispersion $\hat{d}_k'$ of a connected nondirected graph k can be obtained as shown in equation (4) by using d_k'. From equation (10) it can be easily understood that an additional path $d_{ij} = 1$ (or direct edge) increases d_i' by 1, an additional path $d_{ij} = 2$ adds at least 0.5 to d_i', etc. Therefore, vertices connected to large graphs have a higher degree of connectivity. This is generally true, except if for vertex v_i in graph k, and vertex v_j in graph l, with $n_k > n_l$, it can be stated that $d_i' \leqslant d_j'$. The condition for this critical point can be defined as $f_{i,r} < f_{j,r}$ for low values of r. As the network dispersion $\hat{d}_k'$ is the sum of the degree of connectivity d_i' of all vertices v_i in k, critical points can also be defined for the measures of the network dispersion.

Table 1 presents the results of the modified parameter d_i' for the graphs in figure 3. Comparison of table 1 and figure 3 shows that the relative importance of vertices measured by the hierarchy numbers are constant.

Table 1. The modified connectivity parameters d_i' and s_i' per vertex v_i, the hierarchy numbers (order) of the vertices per graph related to d_i' and to s_i', and the network dispersions $\hat{d}_k'$ and $\hat{s}_k'$ of the two graphs as presented in figure 3. High values of d_i' and s_i' correspond to highly connected vertices v_i. For both d_i and s_i, the best connected vertex has the lowest hierarchy number.

v_i	d_i	Order	s_i	Order	v_i	d_i	Order	s_i	Order
Graph 1					Graph 2				
1	3.5	5	0.3	4	7	1	1.5	0.033	1.5
2	4	2.5	0.308	2.5	8	1	1.5	0.033	1.5
3	4	2.5	0.308	2.5	$\hat{d}_2' = 2$,	$\hat{s}_2' = 0.067$			
4	4.5	1	0.317	1					
5	3.5	5	0.133	5.5					
6	3.5	5	0.133	5.5					
$\hat{d}_1' = 23$,	$\hat{s}_1' = 1.5$								

4.3 Geometric approach

In correspondence with the topological approach, the s_i values are suitable measures for the degree of connectivity of vertices in one graph. The same reasoning as for d_i' can be applied for the modification of s_i. The modified s_i' value can then be calculated by

$$s_i' = \sum_{u>0}^{\sigma} \frac{f_{i,u}}{u}, \quad i = 1, 2, \ldots, n. \tag{11}$$

High values of s_i' represent highly connected vertices. The network dispersion $\hat{s}_k'$ can be obtained as shown in equation (4) by using s_k'. Table 1 presents the results of the modified parameter s_i' for the graphs in figure 3.

In the geometric approach, we can also define a critical point between two graphs. The condition for this critical point is: when vertex v_i in graph k, and vertex v_j in graph l, with $n_k > n_l$, fulfil the topological condition $d_i' \leqslant d_j'$, it can be stated that $s_i' \leqslant s_j'$ when $f_{i,u} < f_{j,u}$ for low values of u. The opposite holds when vertex v_i in graph k, and vertex v_j in graph l, with $n_k > n_l$, do not fulfil the topological condition; then $s_i' \leqslant s_j'$ when $f_{i,u} < f_{j,u}$ for low values of u or $f_{i,u} > f_{j,u}$ for high values of u. The geometric conditions for a critical point describe two effects. As s_i' measures the number and the weighted distance of shortest paths, the effect of the one (for example, increasing distance) in s_i' can be nullified by the effect of the other (more direct connections), and vice versa.

5 Simulations

5.1 Effect of size

We considered the effect of size n_k of the vertex set $V(G_k)$ on the parameters d_i' and s_i'. We used three topologically constant graph types which represent different landscape networks: the path graph as a linear landscape network, the completely connected graph as a compact landscape network, and the triangular graph as an extended landscape network (figure 5). For each type we generated a series of g graphs with a systematic increase in the size n_k of the vertex set $V(G_k)$ for each graph k. For the path graph and the completely connected graph, n_k increases by 1 per graph k, starting with $n_1 = 1$. The triangular graphs increase with k (figure 5). All w_{ij} are considered to be equal.

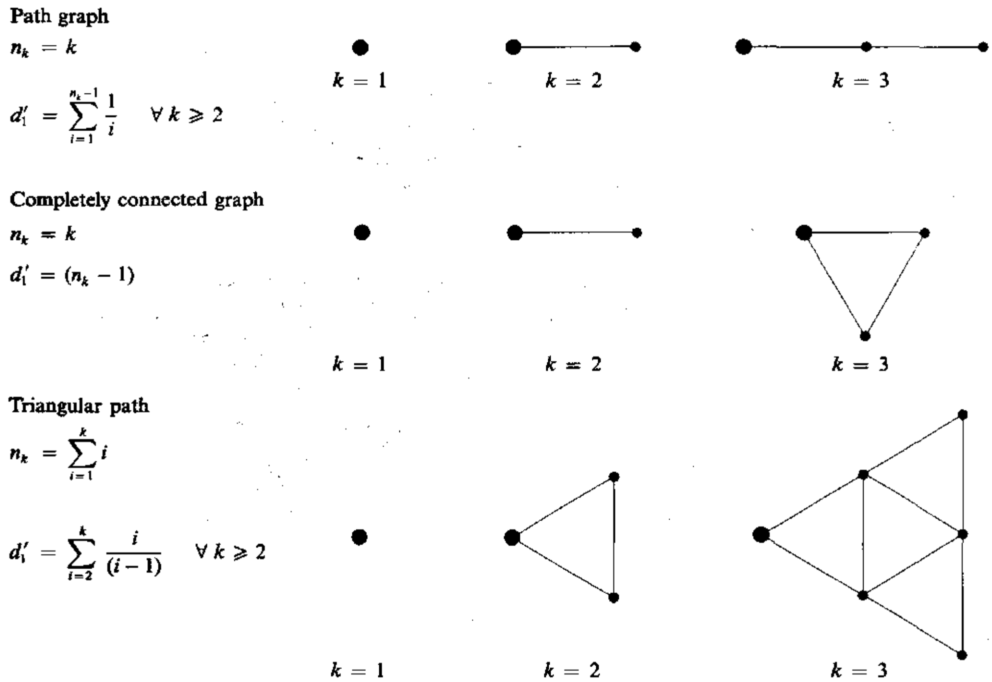


Figure 5. Three graph types with a topologically constant structure: the path graph, the completely connected graph, and the triangular graph. The large vertices (●) are used to illustrate the effect of size on the connectivity parameters. The equations for the size n_k of the vertex set $V(G_k)$ of graph k and for the connectivity parameter d_1' for the indicated vertex are presented. For all three types, we defined $d_1' = 0$ *in graph* $k = 1$.

We calculated d_i' for just one vertex of each graph. In figure 5 the vertices are indicated by large dots. Figure 5 also provides the equations to calculate d_1' for these types. Figure 6 presents the d_1' values of the first vertex of each graph k. The diagrams for s_i' (not presented) showed exactly the same pattern. Figure 7 shows the $\hat{d}_k'$ values for some graphs of the series. The $\hat{s}_k'$ (not presented) values showed the same pattern as the $\hat{d}_k'$ values.

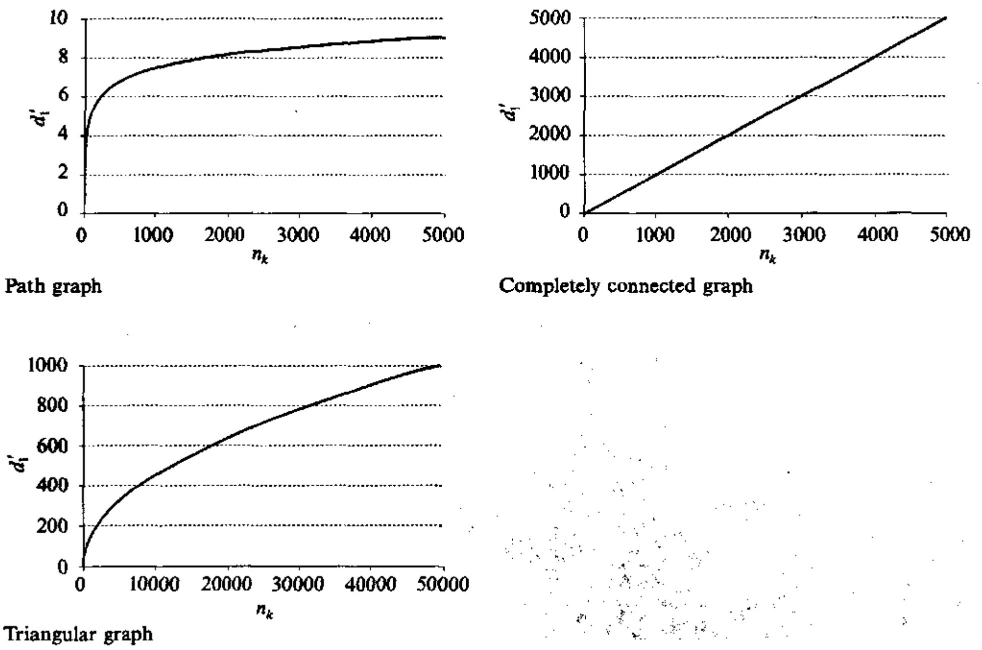


Figure 6. The relationships between the size n_k of the vertex set $V(G_k)$ of each graph k and the connectivity parameter d_1' of the first vertex (see figure 5). For the path graph and the completely connected graph, the number of graphs $g = 5000$. For the triangular graph, $g = 1000$.

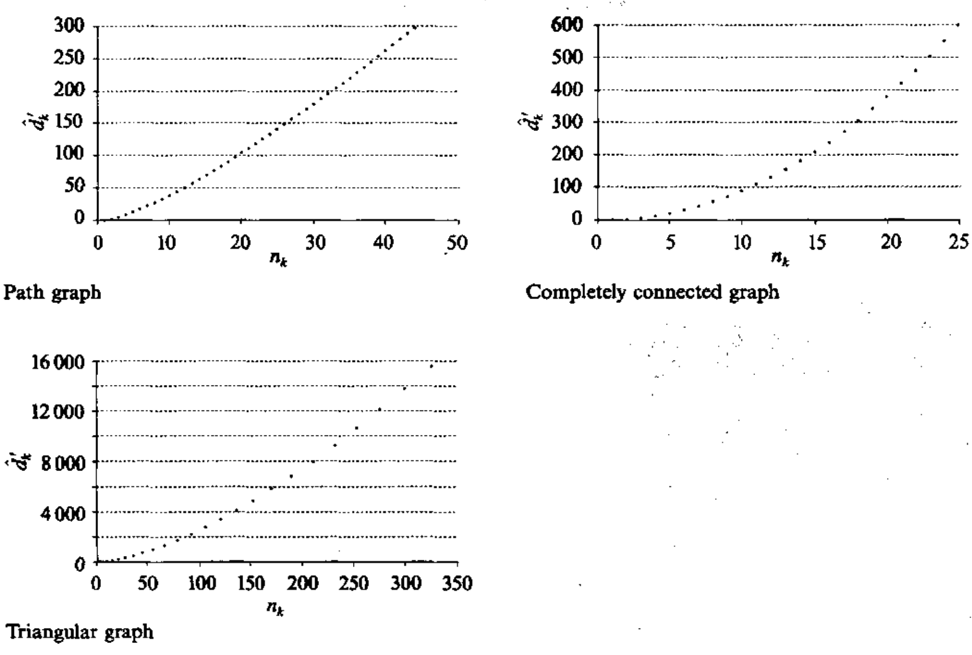


Figure 7. The network dispersion $\hat{d}_k'$ per graph k for three graph types. The graphs systematically increase in size n_k of the vertex set $V(G_k)$ ($g = 44$ for the path graph, $g = 25$ for the completely connected graph, and $g = 25$ for the triangular graph).

5.2 Effect of spatial configuration

We also analyzed the effect of spatial configuration of the network on the parameters. For each graph type, we started with a limited number of differently sized graphs. The spatial configuration of each was independently changed by stretching w_{ij}. This provided a series of differently spaced graphs with constant sizes of the vertex set $V(G_k)$ and edge set $E(G_k)$. Hence, $\hat{c}_k$ and $\hat{d}'_k$ are constant. Because p'_i measures direct edges and s'_i both direct and indirect edges, we looked for the differences between the two parameters.

At the beginning, each graph had $w_{ij} = 10$ for all edges. The graphs changed because of one randomly selected edge which was stretched by $y = 3$ to $w_{ij} = 30$: graph 1 had one randomly stretched edge, graph 2 had one extra randomly stretched edge, etc. This was continued until all edges were stretched. Figure 8 presents the results of the series of stretched graphs. Critical points are indicated.

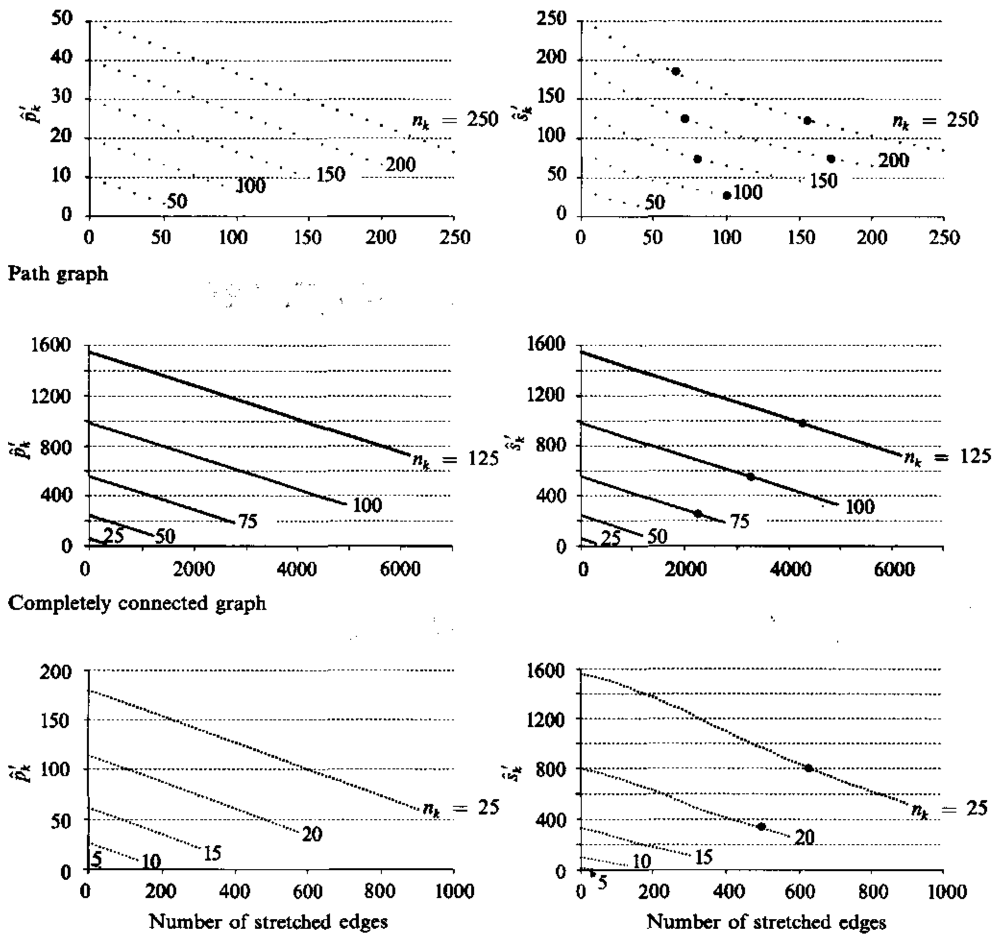


Figure 8. The relationships between the network dispersion $\hat{p}'_k$ (left) and $\hat{s}'_k$ of a limited number of graphs k of the three types, and the number of stretched edges (one randomly selected edge is stretched from $w_{ij} = 10$ to $w_{ij} = 30$). Graph 0 has zero stretched edges, graph 1 has one randomly stretched edge, graph 2 has one extra randomly stretched edge, etc. Critical points are indicated with dots. Critical points occur when the connectivity value of vertices in graph k is lower than for vertices in graph l ($\hat{s}'_k \leqslant \hat{s}'_l$), and $n_k > n_l$.

6 Discussion and conclusion

The graph-theoretic parameters used frequently to quantify the degree of connectivity of network elements cannot be applied to compare the connectivity of these elements when they belong to different networks. This may not be problematic when the objective of the study is to analyze relationships between pattern and function within one (connected) network. However, a modification of the parameters appears to be necessary to compare the degree of connectivity of elements in different networks across regions as well as over time, or to analyze decomposed networks with non-connected subsystems. We extended the existing parameters to overcome this problem. When applying these parameters, one should be aware that the results are dependent upon the number of locations that are accessed by the network or, in other words, by the boundaries of the delimited region. When the degree of connectivity needs to be measured independently of the delimitation of the region, the standardized parameters as derived by either Allen et al (1993) or Teklenburg et al (1993) should be used.

The applications for these parameters can be encountered in comparing the relative position of any location to others over time or among regions, such as shopping centres, recreation facilities, hospitals, cities, and nature reserves. The relationship between variables that represent the function of these locations (for example, number of visitors, telephone calls, passengers by bus, railway, or airline) can be tested against their degree of connectivity. Do differences between regions exist? Have the relationships been changed over time? Other applications in spatial analysis and planning are: as a measure of connectivity or accessibility in other models when such a quantity is required, for example, models of population potential (Pooler, 1987), spatial interaction models, or spatially realistic models for animal populations (Hanski and Gilpin, 1997); to indicate deficiencies of existing systems, to design new configurations, and to evaluate these new arrangements, for example, to address questions such as where facilities or resources may be located in a region so that they are accessible; to illustrate the effect on connectivity of changes in the observed system, for example, the impact of a new town, facility, or train station on relationships with other locations.

We used four types of matrix-based parameters to quantify the degree of connectivity of elements and the dispersion in networks. The parameters take into account only a few aspects of networks: measures for the number and weight of direct and indirect relationships between elements. We did not use other parameters such as the global indices from Kansky (1963), or density or shape indices (Haggett and Chorley, 1969; Selkirk, 1982) because they focus on the network as a whole rather than on individual locations. We also do not consider the size or weights of the locations (see Pooler, 1987; 1995). However, the connectivity parameters can be combined with these characteristics of locations, such as population size and attractiveness.

Each parameter type has certain properties. The first simulation provides insight into the effect of network size on the relevant parameters. Figure 6 shows that size n_k of the vertex set $V(G_k)$ of graph k affects the degree of connectivity of the vertices concerned: when n_k increases, the value d_i' and, therefore, the degree of connectivity of the vertices increases. This agrees with our assumptions. As the graphs per type have the same topological structure, this effect can be explained solely by the increase in the size n_k. We compared the behaviour of d_i' with the original parameter d_i. As can be derived from equation (2), the degree of connectivity d_i of the first vertex of both the path graph and the triangular graph decreases exponentially with an increase in the size n_k of the vertex set $V(G_k)$. As low values of d_i correspond to a high degree of connectivity, this pattern cannot be correct. It implies that vertices connected to vertex sets $V(G_k)$ with a larger size n_k have a lower degree of connectivity (see figure 3). In the completely connected graph, a linear increase in d_i' appears owing to its structure.

This increase is equivalent to that obtained by d_i, because for this graph type it can be stated that $d_i = d_i' = c_i$. For the other vertices in the graphs, we can expect the same effects of size because the graphs are topologically constant.

Network dispersion provides information about the degree of connectivity of individual elements. For example, for matrix **D**, the ratio $\hat{d}_k'/n_k$ is approximately d_i', especially for large graphs. Therefore we calculated $\hat{d}_k'$ per graph k for the three types (figure 7). As can be expected, graph size n_k affects $\hat{d}_k'$. It appears that the increase in $\hat{d}_k'$ becomes linear when the size of large graphs increases. In accordance with the original parameter d_i, $\hat{d}_k$ decreases exponentially with an increase in the graph size n_k; two graphs which are almost equal in size may differ enormously in terms of the degree of connectivity measured by $\hat{d}_k$.

Figure 8 shows the relationships between the parameters $\hat{p}_k'$ and $\hat{s}_k'$ and the number of stretched edges. It is apparent that the diagrams of $\hat{p}_k'$ and $\hat{s}_k'$ for the completely connected graph are equal. In the beginning of the other series, only a few $\hat{s}_i'$ values are affected by stretched edges, especially in extended graphs where alternative shortest-weighted paths are available. When the number of stretched edges increases, two combined effects lead to the nonlinear variation of $\hat{s}_k'$ in the path graphs and the triangular graphs. First, the number of stretched edges increases in each shortest-weighted path s_{ij}. Second, the number of larger s_{ij} in each s_i' increases. The availability of shorter paths then decreases. In comparison with $\hat{p}_i'$ it appears that $\hat{s}_i'$ is sensitive to the multiplicity of edges with changing length. The connectivity parameters $\hat{s}_i'$ and $\hat{s}_k'$ provide refined measures when the spatial configuration is taken into account.

These results are confirmed by other types of graph we investigated: complete bipartite graphs with equal-size subsets and rectangular graphs. Bipartite graphs belong to the type that represents compact landscape networks. The results were comparable with the completely connected graph. The rectangular graph type can be considered to be an extended landscape network. The results were comparable with those of the triangular graph.

We can conclude that the size of the graphs affects the values of the parameters d_i' and s_i'. However, in contrast to d_i and s_i, the degree of connectivity of elements measured by d_i' and s_i' in a variety of differently sized networks can be compared. In accordance with our assumptions about effects of size on the degree of connectivity, the comparisons of the degree of connectivity provide useful results. We can also conclude that the variation in both $\hat{p}_k'$ and $\hat{s}_k'$ and, therefore, in p_i' and s_i' concurs with the variation in the spatial configuration of the network. The s_i' parameter of equation (11) and the resulting $\hat{s}_k'$ parameter can differentiate between differently sized and spaced networks. These parameters can be applied to compare the degree of connectivity of elements in digraphs and in nonconnected subsystems of decomposed networks. In this paper, we assumed a spatial definition of distance that resulted in emphasis on the relevance of the spatial configuration of networks in the analysis. These parameters can also be used for other definitions of space. The interpretation of the results of these applications are beyond the scope of our paper.

Acknowledgements. The authors would like to thank Jason Dykes and Harry Timmermans for their critical comments.

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